

SCORE: ____ / 30 POINTS

GREEN
SHEET Q's
6+

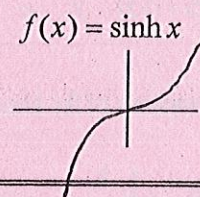
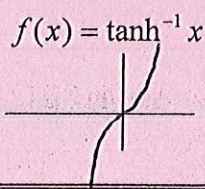
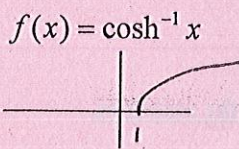
1. No calculators or notes allowed
2. Show proper precalculus-level work (except for graphing questions)
3. Simplify all answers

Rewrite $\operatorname{csch}(3\ln 2)$ in terms of exponential functions and simplify.SCORE: 3 / 3 PTS

$$\operatorname{csch}(3\ln 2) \Rightarrow \frac{2}{e^{3\ln 2} - e^{-3\ln 2}} = \frac{2}{e^{\ln 2^3} - e^{\ln 2^{-3}}} =$$

$$\frac{2}{8 - \frac{1}{8}} = \frac{2}{\frac{63}{8}} = \frac{16}{63}$$

Sketch the general shape and position of the following graphs.
(Don't worry about specific x - or y -coordinates.)

SCORE: 3 / 3 PTSWrite and prove a formula for $\cosh(x+y)$ in terms of $\sinh x$, $\sinh y$, $\cosh x$ and $\cosh y$.SCORE: 6 / 6 PTS

$$\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y \Rightarrow$$

$$\left(\frac{e^x + e^{-x}}{2} \right) \left(\frac{e^y + e^{-y}}{2} \right) + \left(\frac{e^x - e^{-x}}{2} \right) \left(\frac{e^y - e^{-y}}{2} \right) \Rightarrow$$

$$\left(\frac{e^{x+y} + e^{x-y} + e^{-x+y} + e^{-x-y}}{4} \right) + \left(\frac{e^{x+y} - e^{x-y} - e^{-x+y} + e^{-x-y}}{4} \right) =$$

$$\frac{2e^{x+y} + 2e^{-x-y}}{4} = \frac{e^{x+y} + e^{-x-y}}{2} = \cosh(x+y)$$

Prove that $g(x) = \ln(x + \sqrt{x^2 - 1})$ is the inverse of $f(x) = \cosh x$ by simplifying $g(f(x))$.

SCORE: 5 / 5 PTS

You may use any identities that you found in part [1] of the Hyperbolic Functions Supplement without proving them.

$$f^{-1}(f(x)) \Rightarrow g(f(x)) \Rightarrow \ln(\cosh x + \sqrt{\cosh^2 x - 1}) \quad (1)$$

$$\ln(\cosh^2 x + \sqrt{\sinh^2 x}) \Rightarrow \ln(\cosh x + \sinh x) = \ln e^x = x \quad (12) \quad (12) \quad (1)$$

There is an identity involving $\sinh x$ and $\cosh x$ that resembles a Pythagorean identity from trigonometry.

SCORE: ____ / 7 PTS

[a] Write that identity involving $\sinh x$ and $\cosh x$. You do NOT need to prove the identity.

$$\cosh^2 x - \sinh^2 x = 1 \quad (1)$$

[b] Write any two of the three versions of the identity for $\cosh 2x$. You do NOT need to prove the identities.

$$\begin{aligned} \cosh^2 x + \sinh^2 x &= \cosh 2x \\ 2\cosh^2 x - 1 &= \cosh 2x \end{aligned} \quad (2)$$

[c] If $\coth x = 2$, find $\cosh x$ using identities.

You must explicitly show the use of the identities but you do NOT need to prove the identities.

Do NOT use inverse hyperbolic functions nor their logarithmic formulae in your solution.

$$\coth x = 2 \Rightarrow \tanh x = \frac{1}{2} \quad (12)$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x \Rightarrow 1 - \frac{1}{4} = \frac{3}{4} \Rightarrow \operatorname{sech}^2 x = \frac{3}{4} \Rightarrow \operatorname{sech} x = \pm \frac{\sqrt{3}}{2} \quad (12) \quad (12)$$

$$\cosh x = \frac{2\sqrt{3}}{3} \quad (*)$$

$$\operatorname{sech} x = \pm \frac{\sqrt{3}}{2} \quad (*)$$